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МОСКОВСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ
имени М.В.ЛОМОНОСОВА

Вариант А-4

Место проведения Москва
город

ПИСЬМЕННАЯ РАБОТА

Олимпиада школьников Покори Воробьевы горы
наименование олимпиады

по математике
профиль олимпиады

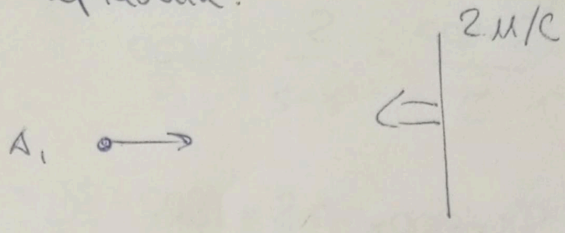
Хоба Артемия Юрьевича
фамилия, имя, отчество участника (в родительном падеже)

Дата

«1» апреля 2024 года

Подпись участника

Терновик.



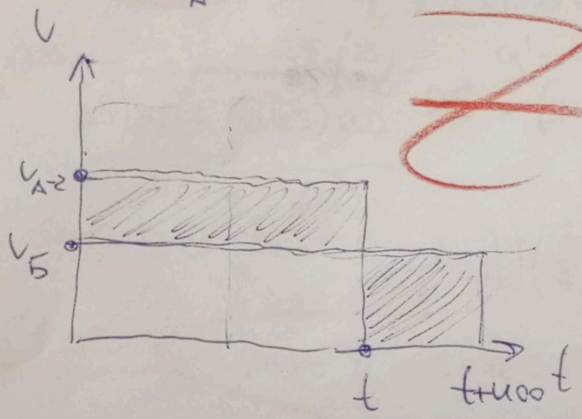
	V	t	S
A1	V_A-2	t	$(V_A-2)t$
Б1	$V_Б$	t+400	$V_Б(t+400)$

V_A-2 $\quad$ $\frac{S}{V_A-2}$ $\quad$ S

$V_Б$ $\quad$ $\frac{S-900}{V_Б}$ $\quad$ S-900

$$(V_A-2)t - V_Б(t+400) = 900$$

$$V_A t - 2t - V_Б t - V_Б \cdot 400 = 900$$



$$\frac{S}{V_A-2} - \frac{S-900}{V_Б} = 900$$

$$SV_Б - SV_A + 2S = 900(V_A-2)V_Б$$

$$\frac{S}{V_A} - \frac{S}{V_Б}$$

$V_Б$

$$\frac{S}{V_A-2} + \frac{S+900}{V_Б} = -400$$

$$SV_Б - V_A S + 900V_A + 2S - 1800 = -400(V_A-2)V_Б$$

	V	t	S
$Д_1$	$x-2$	$\frac{S}{x-2}$	S
$Б_2$	y	$\frac{S-900}{2y}$	$S-900$

$$\frac{S-900}{y} - \frac{S}{x-2} = 400$$

$$Sx - 900x - 2S + 1800 - Sy = 400xy - 800y$$

$$Sx - 900x - 2S + 1800 - Sy < 400xy - 800y$$

$$\cos(\alpha) - \cos(\beta) = -2\sin\frac{\alpha+\beta}{2}\cdot\sin\frac{\alpha-\beta}{2}$$

$$x + \sin x = \frac{\alpha+\beta}{2} \qquad 2x + 2\sin x = \alpha+\beta$$

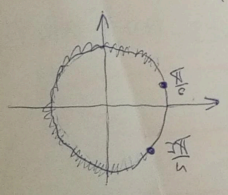
$$x - \sin x = \frac{\alpha-\beta}{2} \qquad 2x - 2\sin x = \alpha-\beta$$

$$36\sin(x+\sin x)\sin(x-\sin x) = 36\cos(2\sin x) - \cos(2x)$$

$$18\left(\cos(2\sin x) - \cos(2x)\right) = (\pi-3)(\pi+3) = \pi^2 - 9 > 0$$

$$18\left(\cos^2(\sin x) - \sin^2(\sin x) - \cos^2(x) + \sin^2(x)\right) < (\pi-3)(\pi+3)$$

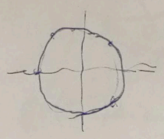
$$18\left(\left(\cos(\sin x) - \cos(x)\right)\left(\cos(\sin x) + \cos x\right) + \left(\sin x - \sin(\sin x)\right)\left(\sin x + \sin(\sin x)\right)\right)$$



Черновик

$$9\left(4\sin(x+\sin x)\sin(x-\sin x) + 1\right) \le \pi^2$$

$$9\left(2\left(\cos(2\sin x) - \cos(2x)\right) + 1\right) \le \pi^2$$



$$\cos\sqrt{2} \quad -\cos\frac{\pi}{2} \quad x = \frac{\pi}{6}$$

$$x = \frac{\pi}{3} \quad \cos\left(1\frac{\pi}{3}\right) - \cos\left(\frac{2\pi}{3}\right)$$

$$x = \frac{-\pi}{4}$$

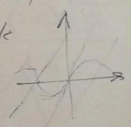
$$\frac{2\cdot 3{,}14}{3} = \frac{6{,}28}{3} = 2$$

$$y = \cos(2\sin x) - \cos(2x) \qquad y' = -\sin(2\sin x) + \sin(2x) = 0$$

$$\sin x < x \Rightarrow \qquad \sin(2\sin x) = \sin(2x)$$

$$2\sin x = \pi - 2x + 2\pi k \qquad 2\sin x = 2x + 2\pi k$$

$$\sin x = \frac{\pi}{2} - x + 2\pi k \qquad \sin x = x + \pi k$$



$$\sin x \neq x$$

$$x = \frac{\pi}{6} \qquad \cos 1 \quad -\cos\frac{\pi}{3} =$$

$$0{,}36 \qquad \frac{1}{2}$$

$S(-2) = |-6+4| - |-6+2| = -2$ S

$S(-1) = |-3+4| - |-3+2|$

$S(0) = 0$

$S(x+3) - 6 \le S(x)$ $\quad S(x+3) - 6 \le S(x+2) - 4$

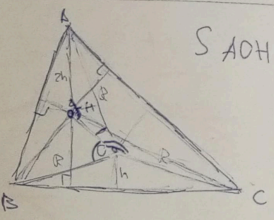
$S(x+2) - 4 \ge S(x)$ $\quad S(x+3) \le S(x+2) + 2$

$S(x+3) \le S(x+2) + 2 \le S(x+1) + 4$

$\le S(x) + 6$

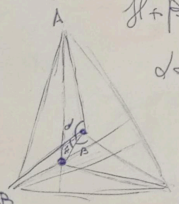
$S(x+2) \ge S(x) + 4$

$S(x+2) \le S(x) + 4$ $\Rightarrow S(x+2) = S(x) + 4$



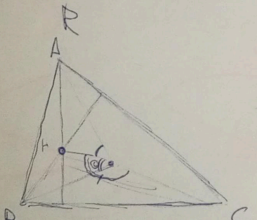
$S_{AOH} = 9$ $\quad S_{BOH} = 5$

$S_{COH} - ?$



$\gamma + \beta = \alpha$

$\alpha = \gamma + \beta$



$S = OH \cdot R \cdot \sin\alpha \cdot \frac{1}{2} = 9$

$S = OH \cdot R \cdot \sin\gamma \cdot \frac{1}{2} = 5$

$S = OH \cdot R \cdot \sin(\gamma + \beta)$

$\frac{\sin\alpha}{\sin\beta} = \frac{9}{5}$

$ON \parallel BH$, $ON = \frac{1}{2}BH$

$CH \parallel OM$, $CH = 2OM$

$AM = MO$

$CN = NA$

$S_{OKC} = 2S_{OHM} \Rightarrow$

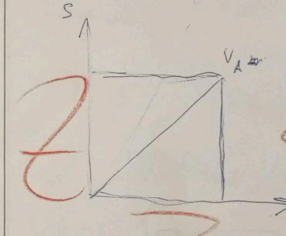
$S_{OHM} = \frac{S_{OHA} + S_{OHB}}{2}$

$S_{OKC} = S_{OHB}$

$S_{OKC} = S_{OKB} + S_{OKA}$

$S_{OKC} = 9 + 5 = 13$

Ответ: 13 или 4.



$V_A \cdot t = S$

$V_A - 2$ $\quad \frac{S}{V_A - 2}$ $\quad S$

$V_Б - 2$ $\quad \frac{S-900}{V_Б - 2}$ $\quad S-900$

V_A $\quad S_1$

$V_Б$ $\quad S_2$

$\frac{S-900}{V_Б - 2} - \frac{S}{V_A - 2} = 400$

$V_A \cdot \frac{S}{V_A - 2} - V_Б \cdot \frac{S-900}{V_Б - 2} - ?$

$(V_A - 2)\cdot t = S$

$(V_Б - 2)\cdot (t+400) = S - 900$

$V_A t - 2t < S$

$V_Б t - 2t + 400 V_Б - 800 < S - 900$

$V_A t - V_Б t - 400 V_Б + 800 < 900$

$V_A t - V_Б \cdot (t+400) < 100$

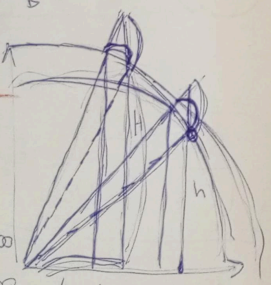
$(V_A - 2)t - (V_Б - 2)(t+400) < 900$

$V_A t - 2t - V_Б t + 2t - 400 V_Б + 800 < 900$

$\frac{S}{V_A - 2}$ $\quad S$

$V_A - 2$

$V_Б - 2$ $\quad \frac{S-900}{V_Б - 2}$ $\quad S-900$

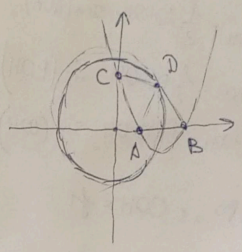


$Ht - h(t+400) > 900$

$Ht - ht - 400h < 900$

Задача 5

$y = x^2 + px + q$



Условия.

$A\{x_1; 0\}$, $B\{x_2; 0\}$,

$C\{\{0; q\}$, $D(a; b)$

$a^2 + b^2 < 2021$

Точка D лежит на окружности с центром $\{0; q\}$ и радиуса $\sqrt{2021}$

$|DA| = \sqrt{(a-x_1)^2 + b^2} = \sqrt{a^2 - 2ax_1 + x_1^2 + b^2} < \sqrt{2021 - 2ax_1 + x_1^2}$

$|DB| = \sqrt{(a-x_2)^2 + b^2} = \sqrt{a^2 - 2ax_2 + x_2^2 + b^2} < \sqrt{2021 - 2ax_2 + x_2^2}$

$|DC| < \sqrt{a^2 + (b-q)^2} < \sqrt{a^2 + b^2 - 2bq + q^2} < \sqrt{2021 - 2bq + q^2}$

$|DA| < |DB| \Rightarrow 2021 - 2ax_1 + x_1^2 < 2021 - 2ax_2 + x_2^2$

$2a(x_2 - x_1) - (x_2 - x_1)(x_2 + x_1) < 0$

$(x_1 - x_2)(2a - x_1 - x_2) < 0$

$x_1 - x_2 < 0$ — невозможно по усл.

$2a - x_1 - x_2 < 0$

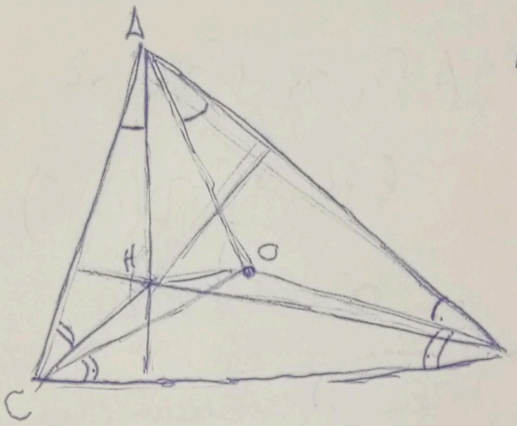
$a < \frac{x_1 + x_2}{2} = \frac{-p}{2}$ (по т. Виета)

$\Downarrow$

$b < \sqrt{2021 - \frac{p^2}{4}}$

Тогда $|DC| < \sqrt{2021 - 2q\sqrt{2021 - \frac{p^2}{4}} + q^2}$

Задача 4



$AO = OB = OC = R$

$S_{AOH} = \frac{1}{2} AO \cdot OH \cdot \sin(AOH)$

$S_{BOH} = \frac{1}{2} BO \cdot OH \cdot \sin(BOH)$

$S_{COH} = \frac{1}{2} CO \cdot OH \cdot \sin(COH)$

Пусть $\angle AOH = \alpha$, $\angle BOH = \beta$, $\angle COH = \gamma$

Тогда $\frac{\sin\alpha}{\sin\beta} = \frac{a}{5}$

Задача 6 $|2[\log a] + 1|^x = [\log a]^2 + 2$

$[\log a] = k; \; k \in Z \quad k \neq 0$

$|2k+1|^x = k^2 + 2$

$$\left(\frac{H}{t}+2\right)t-\left(\frac{h}{t}+2\right)(t+400)$$

$$H+2t-h-2t+\frac{400h}{t}-800=x$$

$$H-h=\frac{800+400h}{t}$$

$$\left(\frac{H}{t}+2\right)t-Ht\cdot\frac{1}{2}-\left(\frac{h}{t}+2\right)(t+400)-ht\cdot\frac{1}{2}$$

$$H+2t-Ht-h-2t-\frac{400h}{t}-800+ht=2x$$

$$H-h-\frac{400h}{t}+100=2x$$

$$\frac{800}{t}+100=2x$$

$$\begin{array}{r} 1012 \\ \times\ \ 4 \\ \hline 4048 \\ +\ \ 9 \\ \hline 4057 \end{array}$$

$$V_1\cdot t-V_2\cdot(t+400)=800$$

$$(V_1+2)t-(V_2+2)(t+400)=?$$

$$t(V_1-V_1-2)-(t+400)(V_2-V_2-2)=800-x$$

$$-2t+2t+$$

$[\operatorname{ctg} a] = k$

$|2k+1|^x = k^2+2$

$x=1$: $k^2-2k+1=0$ $k=1$

$2k+1 \geq 0$

$x=2$ $4k^2+4k+1=k^2+1$

$2k+1\geq 0$ $3k^2+4k=0$

$t(3k+4)=0$

$x=0$ $1=k^2+2$ нет.

$\frac{\pi}{4}$ $\cos(\sqrt{2})$ $-$ $\cos\left(\frac{\pi}{2}\right)$

$\frac{\pi}{2}$ v $\sqrt{2}$

(1, 6) > 1,4 $\sin\frac{2\pi}{3} = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

Чистовик:

Задача 1

Пусть скорость $А_1 = V_1+2$, $Б_2 = V_2+2$, тогда

$$\begin{cases} V_1 \cdot t - V_2 \cdot (t+400) = 900 \\ (V_1+2)t - (V_2+2)(t+400) = x \end{cases}$$

Вычтем из верхнего нижнее, тогда:

$t(V_1 - V_1 - 2) - (t+400)(V_2 - V_2 - 2) = 900 - x$

$-2t + 2t + 800 = 900 - x \Rightarrow x = 100$

$\Rightarrow$ Первый (А-1) пролетит на 100 метров больше.

Задача 2.

$$\begin{cases} f(x+3) - 6 \leq f(x) \\ f(x+2) - 4 \geq f(x) \end{cases} \Rightarrow f(x+2) - 4 \geq f(x+3) - 6$$

$\Downarrow$

$f(x+3) \leq f(x+2) + 2 \leq f(x+1) + 4 \leq f(x) + 6$

$\Downarrow$

$f(x+2) \leq f(x) + 4$

Но из условия:

$f(x+2) - 4 \geq f(x)$

$f(x+2) \geq f(x) + 4$ $\rightarrow$ Следовательно:

$f(x+2) = f(x) + 4$

$f(0) = 9 \Rightarrow f(2) = 13$, $f(4) = 17$ ..... $f(2024) = 9 + 4 \cdot 1012 = 4057$. Отв: 4057.

Задача 3 Числовик.

$36\sin(x+\sin x)\sin(x-\sin x)+9=\pi^2$

$9\left(4\sin(x+\sin x)\sin(x-\sin x)+1\right)=\pi^2$

$\sin(x+\sin x)\sin(x-\sin x)=\frac{1}{2}\left(\cos(2\sin x)-\cos(2x)\right)$

$\Rightarrow 9\left(2\left(\cos(2\sin x)-\cos(2x)\right)+1\right)=\pi^2$

$\cos(2\sin x)-\cos 2x=\frac{\pi^2}{18}-\frac{1}{2}$

~~$\sin x < x$ при $x>0$, тогда $2\sin x < 2x$~~

$a\in[-1,1]$

$1-2\sin^2 x$ $\quad \cos 2a-1+2a^2=$

$=\frac{\pi^2}{18}-\frac{1}{2}$

$\frac{1}{2}-1+\frac{\pi^2}{18}=$

$=\frac{\pi^2}{18}-\frac{9}{18}$ верно

$x=\arcsin\frac{\pi}{6}+\pi n,\ n\in\mathbb{Z}$

$x=-\arcsin\frac{\pi}{6}+\pi n,\ n\in\mathbb{Z}$

Понадей: $\arcsin\frac{\pi}{6}$

~~$1-\arcsin\frac{\pi}{6}$~~

~~$1+\arcsin\frac{\pi}{6}<\frac{\pi}{6}$~~

Сумма: $2\pi+\arcsin\frac{\pi}{6}$

$f'(a)>0$ при $a>0$

$f'(a)<0$ при $a<0$

$\Rightarrow f(a)\uparrow$ при $a>0$

$f(a)\downarrow$ при $a<0$

$\Rightarrow$ ур. $a=\frac{\pi}{6}$ – верно

$a=-\frac{\pi}{6}$ – верно

$-2ax_2+x_2^2=-2bq+q^2$

$x_2^2-2ax_2+2bq-q^2\le 0$

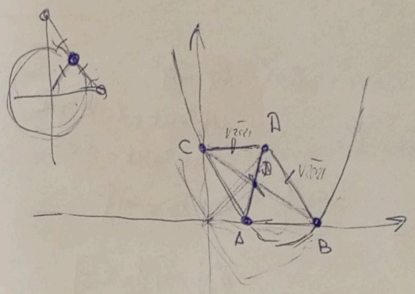
Терновик.

x^2+px+q

$C=\{0;q\}$

$A=\{x_1;0\}$

$B=\{x_2;0\}$



$D(a;b)\quad a^2+b^2\le 2021$

$DA=\sqrt{(a-x_1)^2+(b-0)^2}=\sqrt{a^2-2ax_1+x_1^2+b^2}$

$=\sqrt{2021-2ax_1+x_1^2}$

$DB=\sqrt{(a-x_2)^2+b^2}=\sqrt{2021-2bx_1+x_1^2}$

$DC=\sqrt{(a-0)^2+(b-q)^2}=\sqrt{a^2+b^2-2bq+q^2}=$

$=\sqrt{2021-2bq+q^2}$

$2021-2ax_1+x_1^2\le 2021-2bx_1+x_1^2$

$x_1(2b-2a)\le 0 \Leftrightarrow a\le b$ или $x_1\le 0$

$\downarrow$

$q\le 0$ $D\in\sqrt{2021}$

$x_1x_2=-p$ $\le b$

$x_1x_2=q$ $\ne p$

$2021 - 2ax_2 + x_2^2 \le 2021 - 2bq + q^2$

$2021 - 2ax_1 + x_1^2 \le 2021 - 2bq + q^2$

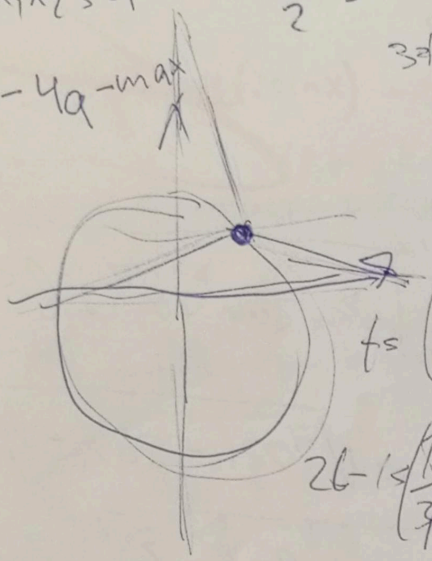
$4k^2+4k+1 \le k^2+2$

$3k^2+4k-1 \le 0$

$x_1 + x_2 = -p$

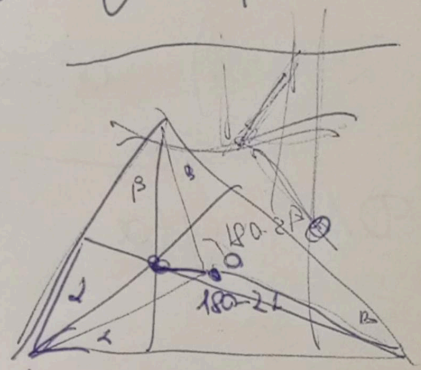
$x_1 x_2 = q$

$p^2 - 4q - \max$



$t = \left(\frac{1}{3}\right)^2 + 1$

$2b - 1 \le \frac{11}{3}$



$\sin(\alpha+\beta) = \sin\alpha \cdot \cos\beta \cdot \sin\beta\alpha$

$\frac{\sin\alpha}{\sin\beta} = \frac{a}{5}$ сим мон acy

$\left(\frac{x_1}{3}\right)^2$

$\sin(180 - \alpha - \beta) =$

$|2k+1| = k^2+2$

$\sin\alpha+\beta$

$2k+1 \ge 0$

$2k^2 - 2k + 1 \le 0 \quad k \in \emptyset$

$\alpha + \beta - \gamma = 180^\circ$

$2k+1 \le 0$

$x \le 0$

$-2k-1 \le k^2+2$

$k^2+2k+3 \ge 0$

$D = \ldots$

$1 < k^2+2$