Олимпиада ПВГ
2016

98-83-76-39
(115.1)

МОСКОВСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ
имени М.В.ЛОМОНОСОВА

г. Уфа

Вариант 3-2

ПИСЬМЕННАЯ РАБОТА

Олимпиада школьников Покори Воробьевы горы

по математике

Ботвинниковой Анастасии Антоновны
фамилия, имя, отчество (в родительном падеже)

Дата
«13» марта 2016 года

Подпись участника

98-83-76-39 (115.1)

ЧИСТОВИК.

Олимпиада ПВГ 2016

80 восемьдесят ВКЛ (Вязьмин)

(Пришин)

1	2	3	4	5
+	−	+	+	+

№1. +

$8x^2+y^3=2016$, $x, y \in N$.

$\left.\begin{array}{l} 8x^2 \vdots 8 \\ 2016 \vdots 8 \end{array}\right\} \Rightarrow y^3 \vdots 8 \Rightarrow y \vdots 2$, пусть $y=2a$, $a \in N$.

$8x^2+8a^3=2016$

$x^2+a^3=252$.

Запишем таблицу кубов меньших 252.

a	1	2	3	4	5	6	7
a^3	1	8	27	64	125	216	343

$1 \le a \le 6$

$252-1=251$ – не кв.

$252-8=244$ – не кв.

$252-27=225=15^2$ – кв.

$252-64=188$ – не кв.

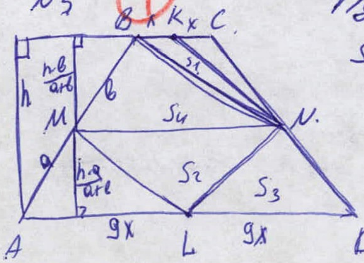
$252-125=127$ – не кв.

$252-216=36=6^2$ – кв.

$\left[\begin{array}{l} \begin{cases} a=3 \\ x=15 \end{cases} \\ \begin{cases} a=6 \\ x=6 \end{cases} \end{array}\right.$ $\left[\begin{array}{l} \begin{cases} y=6 \\ x=15 \end{cases} \\ \begin{cases} y=12 \\ x=6 \end{cases} \end{array}\right.$

Ответ: $(6;12), (15;6)$

№3 +



Пусть $AM=a$; $BM=b$; $\frac{a}{b}=t$; $S_{BKC}=S_1$; $S_{MNL}=S_2$; $S_{MBN}=S_4$; $S_{AML}=S_3$; $S_{ABCD}=S$. $S_1=\frac{1}{2}\frac{h \cdot b}{a+b} \cdot x$

$\left.\begin{array}{l} S_{KCN}=S_{BKN}=S_1 \\ S_{AML}=S_{LND}=S_3 \end{array}\right\}$ по равным основаниям и высоте.

$S_2=\frac{1}{2}\frac{h \cdot a}{a+b} \cdot MN$; $S_4=\frac{1}{2}\frac{h \cdot b}{a+b} \cdot MN=\frac{1}{2}\frac{h \cdot MN}{a+b} \cdot a \cdot \frac{b}{a}=$

$=S_2 \cdot \frac{b}{a}$; $S=\frac{1}{2}(18x+2x) \cdot h=10xh$; $S_3=\frac{1}{2} 9x \cdot \frac{h \cdot a}{a+b}$

$S=2S_1+2S_3+S_4+S_2=2 \cdot \frac{1}{2} \cdot \frac{h \cdot b \cdot x}{a+b}+2 \cdot \frac{1}{2} \frac{9 \cdot x \cdot h \cdot a}{a+b}+S_2+\frac{S_2 b}{a}=$

$=\frac{xh(b+9a)}{(a+b)}+\frac{S_2(a+b)}{a}$

$\frac{S_2(a+b)}{a}=10xh-\frac{xh(b+9a)}{(a+b)}=\frac{xh(10a+10b-b-9a)}{a+b}=\frac{xh(a+9b)}{a+b}$

$9^{f(t)} \in ([9^{-\frac{1}{2}}; 9^{\frac{1}{2}})$

Чистовик.

$9^{f(t)} \in \left[\frac{1}{3}; 3\right]$

$\frac{1}{3} \leq 9^{\frac{6x+3}{4x^2+4x+10}} \leq 3$

$b \in \left(-\infty; \frac{1}{3}\right]$

$a \in (3; +\infty)$

Ответ: $a \in (3; +\infty)$;
$b \in \left(-\infty; \frac{1}{3}\right]$.

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Ответ: $a \in (3; +\infty)$;
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ЧИСТОВИК.

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+	−	+	+	+

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№1. +

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$252 - 27 = 225 = 15^2$ – уд.

$252 - 64 = 188$ – не уд.

$252 - 125 = 127$ – не уд.

$252 - 216 = 36 = 6^2$ – уд.

$\left[\begin{array}{l} \begin{cases} a = 3 \\ x = 15 \end{cases} \\ \begin{cases} a = 6 \\ x = 6 \end{cases} \end{array}\right.$ $\left[\begin{array}{l} \begin{cases} y = 6 \\ x = 15 \end{cases} \\ \begin{cases} y = 12 \\ x = 6 \end{cases} \end{array}\right.$

Ответ: $(6; 12), (15; 6)$

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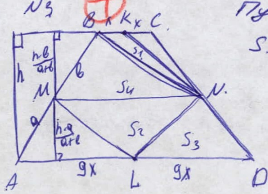

Пусть $AM = a$; $BM = b$; $\frac{a}{b} = t$; $S_{BKC} = S_1$; $S_{MNL} = S_2$;

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$= S_2 \cdot \frac{b}{a}$; $S = \frac{1}{2}(18x + 2x) \cdot h = 10xh$; $S_3 = \frac{1}{2} 9x \cdot \frac{h \cdot a}{a+b}$

$S = 2S_1 + 2S_3 + S_4 + S_2 = 2 \cdot \frac{1}{2} \cdot \frac{h \cdot b \cdot x}{a+b} + 2 \cdot \frac{1}{2} \cdot \frac{9 \cdot x \cdot h \cdot a}{a+b} + S_2 + \frac{S_2 b}{a} =$

$= \frac{xh(b + 9a)}{(a+b)} + \frac{S_2(a+b)}{a}$

$\frac{S_2(a+b)}{a} = 10xh - \frac{xh(b+9a)}{(a+b)} = \frac{xh(10a + 10b - b - 9a)}{a+b} = \frac{xh(a+9b)}{a+b}$

в итоге:

Черновик

$x \in \left[-\frac{\pi}{2}+2\pi n; -\frac{\pi}{12}+2\pi n\right] \cup \left[\frac{7\pi}{12}+2\pi n; \pi+2\pi n\right], n \in \mathbb{Z}.$

Ответ: $x \in \left[-\frac{\pi}{2}+2\pi n; -\frac{\pi}{12}+2\pi n\right] \cup \left[\frac{7\pi}{12}+2\pi n; \pi+2\pi n\right], n \in \mathbb{Z}.$

№2. ⊖

$S_{1-5} = \frac{b_1(q^5-1)}{(q-1)} = 242$

$S_{6-10} = \frac{b_1 q^5(q^5-1)}{(q-1)} = 58806$

$\frac{S_{6-10}}{S_{1-5}} = q^5 = \frac{58806}{242}$ $\quad q = \sqrt[5]{\frac{58806}{242}} = \frac{\sqrt[5]{58806 \cdot 242^4}}{242}$

$S_{1-6} = \frac{b_1(q^6-1)}{q-1} = \frac{S_{1-5} \cdot (q^6-1)}{(q^5-1)} = \frac{242 \cdot (q^6-1)}{\left(\frac{58806}{242}-1\right)} =$

$= \frac{242^2 \cdot (q^6-1)}{58564} = \frac{242^2}{58564} \cdot \left(\frac{58806}{242} \cdot \sqrt[5]{\frac{58806}{242}} - 1\right) =$

$= \frac{242^2}{58564} \cdot \frac{(58806\sqrt[5]{58806} - 242\sqrt[5]{242})}{242\sqrt[5]{242}} = \frac{\sqrt[5]{242^4}\left(\sqrt[5]{58806^6} - \sqrt[5]{242^6}\right)}{58564} =$

$= \frac{\sqrt[5]{242^4 \cdot 58806^6} - 242^2}{58564} = \frac{\sqrt[5]{2^4 \cdot 11^8 \cdot 2^6 \cdot 3^{36} \cdot 41^6} - 242^2}{58564} =$

$= \frac{2^2 \cdot 11 \cdot 3^7 \cdot 41 \sqrt[5]{11^3 \cdot 3 \cdot 41} - 2^2 \cdot 11^4}{58564}$

не дорешено

№5. ⊕

$6 \le 9^{\frac{6x+3}{4x^2+4x+10}} < a.$

$f(t) = \frac{6x+3}{4x^2+4x+10} = \frac{3(2x+1)}{(2x+1)^2+9} = \frac{3t}{t^2+9}$

$f' = \frac{3(t^2+9)-2t \cdot 3t}{(t^2+9)^2} = \frac{3(3-t)(3+t)}{(t^2+9)^2}$

$- \quad -3 \quad + \quad 3 \quad -$

min max

$9^{\frac{3t}{t^2+9}} > 0 \quad 9^{\frac{3t}{t^2+9}} \to 1$

$f(t)_{min} = \frac{-9}{18} = -\frac{1}{2}$ $\quad f(t)_{max} = \frac{9}{18} = \frac{1}{2}$

$S_2 = \frac{xh \cdot (a^2+9ab)}{(a+b)^2}$ Чистовик

$S_1+S_2 = \frac{xha(a+9b)}{(a+b)^2} + \frac{xhb}{2(a+b)} = \frac{xh(2a^2+18ab+ab+b^2)}{2(a+b)^2} =$

$= \frac{xhb^2(2\frac{a^2}{b^2}+19\frac{a}{b}+1)}{2b^2(\frac{a}{b}+1)^2} = \frac{xh}{2} \cdot \frac{(2\frac{a^2}{b^2}+19\frac{a}{b}+1)}{(\frac{a}{b}+1)^2}$

$f(S_1,S_2) = \frac{hx}{2} \cdot \frac{2t^2+19t+1}{(t+1)^2}$

$f' = \frac{xh}{2} \cdot \frac{((4t+19)(t+1)^2 - 2(t+1)(2t^2+19t+1))}{(t+1)^4} =$

$= \frac{xh}{2} \cdot \frac{(t+1)(4t^2+19t+4t+19-4t^2-38t-2)}{(t+1)^4} =$

$= \frac{xh}{2} \cdot \frac{(17-15t)}{(t+1)^3}$

$+ \quad \frac{17}{15} \quad -$ max

$f' = 0$

$t = \frac{17}{15}$

$\frac{17}{15} = \frac{a}{b} = \frac{AM}{BM}$ Ответ: $\frac{17}{15}$

№4 +

(*) $\sin 2x \le 0$

$\sin x \cdot \cos x \le 0$

$x \in [-\frac{\pi}{2}+\pi n; \pi n]$

$a = \cos^3 x + (\sin x + \cos x)\sin x \cdot \cos x + \sin^3 x < \sqrt{-\sin 2x}$

$a = (\cos x + \sin x)(\cos^2 x + \sin^2 x - \sin x \cdot \cos x) + (\sin x + \cos x)\sin x \cdot \cos x =$

$= (\cos x + \sin x)(1 - \sin x \cdot \cos x + \sin x \cdot \cos x) = \cos x + \sin x$

$\cos x + \sin x < \sqrt{-\sin 2x}$

I сл. $\cos x + \sin x \le 0$

$\sin(x+\frac{\pi}{4}) \le 0$

$-\pi + 2\pi n \le x + \frac{\pi}{4} \le 2\pi n$

$-\frac{5\pi}{4} + 2\pi n \le x \le -\frac{\pi}{4} + 2\pi n$

с учетом (*)

$x \in [\frac{3\pi}{4}+2\pi n; \pi + 2\pi n] \cup [-\frac{\pi}{2}+2\pi n; -\frac{\pi}{4}+2\pi n]$

II сл. $\cos x + \sin x \ge 0$

$\cos x + \sin x < \sqrt{-\sin 2x}$

$\cos^2 x + \sin^2 x + 2\sin x \cdot \cos x =$

$= 1 + \sin 2x < -\sin 2x$

$\sin 2x < -\frac{1}{2}$

$-\frac{5\pi}{6} + 2\pi n < 2x < -\frac{\pi}{6} + 2\pi n$

$-\frac{5\pi}{12} + \pi n < x < -\frac{\pi}{12} + \pi n$ (1)

$\cos x + \sin x \ge 0$ $\sin(x+\frac{\pi}{4}) \ge 0$

$2\pi n \le x + \frac{\pi}{4} \le \pi + 2\pi n$

$-\frac{\pi}{4} + 2\pi n \le x \le \frac{3\pi}{4} + 2\pi n$ (2)

учитывая (1), (2), (*):

$x \in [-\frac{\pi}{4}+2\pi n; -\frac{\pi}{12}+2\pi n) \cup (\frac{7\pi}{12}+2\pi n; \frac{3\pi}{4}+2\pi n]$

Черновик

$\cos x + \sin x < \sqrt{-8\sin 2x}$

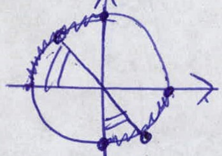
I сл.

$\cos x < -\sin x$

$\cos x \cdot \sin x < 0$

$\frac{3\pi}{4} = +\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} < \sqrt{-(-1)} = 1.$

$0 < 1.$ – уд.



$x \in \left[\frac{3\pi}{4}+2\pi n; \pi+2\pi n\right] \cup \left[-\frac{\pi}{2}+2\pi n; -\frac{\pi}{4}+2\pi n\right]$

II сл.

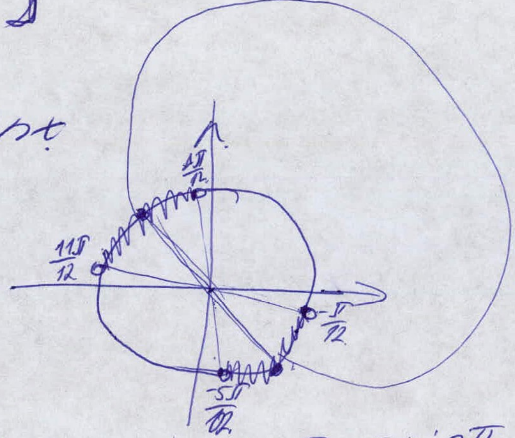
$\cos x + \sin \geq 0 \quad \cos x \geq -\sin x$

$1+\sin 2x < -\sin 2x$

$\sin 2x < -\frac{1}{2}$

$-\frac{5\pi}{6}+2\pi n < 2x < -\frac{\pi}{6}+2\pi n$

$-\frac{5\pi}{12}+\pi n < x < -\frac{\pi}{12}+\pi n$



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~~$\cos x + \sin x < \sqrt{-8\sin 2x}$~~

~~$1+\sin 2x < -\sin 2x$~~

$x \in \left[-\frac{\pi}{2}; -\frac{\pi}{12}+2\pi n\right) \cup \left(\frac{7\pi}{12}+2\pi n; \pi+2\pi n\right]; n \in \mathbb{Z}$

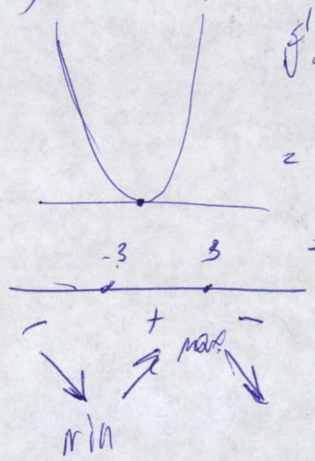
5) $\frac{6x+3}{4x^2+4x+10} = \frac{3(2x+1)}{(2x+1)^2+9} = \frac{3t}{t^2+9}$

$f' = \frac{3(t^2+9) - 2t \cdot 3t}{(t^2+9)^2} = \frac{3t^2+27-6t^2}{(t^2+9)^2} = \frac{-3t^2+27}{(t^2+9)^2} = \frac{3(3-t)(3+t)}{(t^2+9)^2}$

$-2 \quad -1 \quad 0 \quad 1 \quad 2$

$-\frac{6}{13} \quad -\frac{3}{10} \quad 0 \quad \frac{3}{10} < \frac{6}{13}$

$39 < 60$



$242 \mid 2$
$121 \mid 11^2$

$242 = 2 \cdot 11^2$

$58806 = 2 \cdot 3^6 \cdot 41$

58806	2
29403	3
9801	3
3267	3
1089	3
363	3
123	3
41	

Черновик.

$x^2+a^3=252.$

$8x^2+y^3=2016$

$y^3 \vdots 8 \quad y \vdots 2$

2016 | 2
1008 | 2.
504 | 2.
252 | 2
126 | 2
63 | 9
7 | 7.

$2016=2^5 \cdot 3^2 \cdot 7.$

$2016 : 8 = 252$

$2 \cdot 8 = 16 \equiv -2.$

x	x^3	x^2	$8x^2$
0	0	0	0.
1	1	1	8.
2	8	4	$32 \equiv 5.$
3	27/0	9/0	0
4	1	16/7	$56 \equiv 2$
5	8.	25/6	$48 \equiv 3$
6	0	0	0.
7.	1.	49/4	5.
8	8.	1.	8.

$125-90=35-27=8.$

$49 \cdot 7=280+63=343$

$393-333=10 \equiv 1.$

$4 \cdot 7=28 \equiv 1.$

$8 \equiv 8$

$64 \equiv 1$

$8 \equiv -1.$

$8^3 \equiv -1.$

$63 \cdot 8+1 \cdot 8 \equiv 8.$

$36 \cdot 6=$
$=180+36=$
$=216$

$252-169=83$

$252-64=188$

ост	$8x^2$	y^3
0	0	0
1	8	1
2	5	8
3	0	0
4	2	1
5	3	8
6	0	0
7	5	1
8	8	8

x	x^2	x^3
1	1	1
2	4	8
3	9	27.
4.	16	64.
5.	25	125.
6	36	216
7	49	343
8	64.	
9	81.	
10	100	
11	121.	
12	144	
13	169	
14	196	
15	225.	
16	256.	

$252-225=27.$

$252-216=36.$

$252-196=56$ -н.

$252-169=83$ -н.

$252-144=108$ -н.

$252-121=131$ -н.

~~252-128=124~~ -н.

к $252-125=127$

$252-100=152$ -н.

$252-81=171$ -н.

$252-64=188$ -н.

$252-49=203$ -н.

$252-36=216$ - было.

$252-25=227.$

$252-16=236$

$252-9=243=81 \cdot 3=3^5.$

$252-4=248$

$252-1=251.$

$252=15^2+3^3$
$252=6^2+6^3$

58806 | 242.
−484. | 2
104

№1.

$y \vdots 2.$ $\quad y=2a.$ $\quad x^2+a^3=252.$

a	1	2	3	4	5	6	7
a^3	1	8	27	64.	125	216	343

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$252-216=36=6^2$ - ув

~~252~~

$\begin{cases} a=3 \\ x=15, \end{cases}$ $\begin{cases} y=6 \\ x=15. \end{cases}$

$\begin{cases} a=6 \\ x=6 \end{cases}$ $\begin{cases} y=12 \\ x=6 \end{cases}$

$58806-242=58564$

Черновик

$$S_n = \frac{b_1(q^n-1)}{q-1}$$

$$S_5 = \frac{b_1(q^5-1)}{q-1} = 242$$

$$\frac{S_{6-10}}{S_5} = q^5$$

$$S_{6-10} = \frac{b_1 \cdot q^5(q^5-1)}{q-1} = 58806$$

58806 : 242; 484; 1080; 968; 1126; 968; 158; 245

$$b_1 + b_1q + b_1q^2 + b_1q^3 + b_1q^4 = \frac{b_1(q-1)(1+q+q^2+q^3+q^4)}{q-1} = \frac{b_1(q^5-1)}{q-1}$$

$$b_1q^5(1+q+q^2+q^3+q^4) = \frac{b_1 \cdot q^5(q^5-1)}{(q-1)}$$

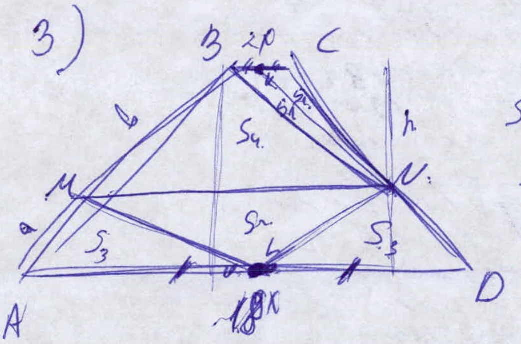
29403 : 121; 242; 510; 484; 263; 242; 21; 243

$$q^5 = 242\frac{21}{121}$$

$$\frac{1}{q^5} = \frac{242}{58806} = \frac{121}{29403}$$

$$S_6 = \frac{b_1(q^6-1)}{(q-1)}\ ?$$

3)



$S_1 + S_2 = \max$

$$S_3 = \frac{1}{2} \cdot 9x \cdot \frac{h \cdot a}{a+b} \qquad S_1 = \frac{1}{2} x \cdot \frac{h \cdot b}{a+b}$$

$$2S_3 = \frac{(9x \cdot h) \cdot a}{a+b} \qquad 2S_1 = (x \cdot h) \cdot \frac{b}{a+b}$$

$$S_4 = \frac{1}{2}\frac{h \cdot b}{a+b} \cdot MN = \frac{1}{2}\frac{h \cdot MN \cdot a}{(a+b)} \cdot \frac{b}{a} = S_2 \frac{b}{a}$$

$$S_2 = \frac{1}{2} h \frac{a}{(a+b)} \cdot MN$$

$$\frac{1}{2}(18x + 2x) \cdot h = 10xh = 2S_1 + 2S_3 + S_2 + S_4 = \cancel{xh \cdot \frac{b}{a+b}} + \frac{9xh \cdot a}{a+b} +$$

$$= 2S_1 + 9x \cdot h \cdot \frac{a}{a+b} + S_2 + S_2 \cdot \frac{b}{a} = \frac{xh \cdot b}{a+b} + \frac{9xh \cdot a}{a+b} + \frac{S_2 \cdot (a+b)}{a} =$$

$$= \frac{xh(b+9a)}{a+b} + \frac{S_2(a+b)}{a} = 10xh$$

$$\frac{S_2(a+b)}{a} = xh\left(10 - \frac{b+9a}{a+b}\right) = xh\left(\frac{10a+10b-b-9a}{a+b}\right) = \frac{xh(a+9b)}{a+b}$$

$$S_2 = \frac{xh\,a(a+9b)}{(a+b)^2} \qquad S_2 + S_1 = \frac{xh\,a(a+9b)}{(a+b)^2} + \frac{xh \cdot b}{2(a+b)} = \frac{xh(2a^2+18ab+ab+b^2)}{2(a+b)^2} =$$

$$= \frac{xh\,b^2(2\frac{a^2}{b^2} + 19\frac{a}{b} + 1)}{2b^2(\frac{a}{b}+1)^2} = \frac{xh(2\frac{a^2}{b^2} + 19\frac{a}{b} + 1)}{2(\frac{a}{b}+1)^2} = \frac{xh(2t^2 + 19t + 1)}{2(t^2+2t+1)}$$

$\left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2} = \frac{4(4t+19)(t+1)^2 - 2(t+1)(2t^2+19t+12)}{(t+1)^4} =$

$= \frac{(t+1)(4t^2+19t+4t+19-4t^2-38t-2)}{(t+1)^3} =$

$= \frac{17-15t}{(t+1)^3} = 0. \quad t = \frac{17}{15}.$

$\frac{17}{15}$ max

Ответ: $\frac{17}{15}$.

Черновик.

4.) $\sin 2x \leq 0$

$\sin x \cdot \cos x \leq 0$

$(\cos x+\sin x)(\cos^2 x - \cos x\cdot\sin x + \sin^2 x) + (\cos x + \sin x)\cos x\cdot\sin x =$

$= (\cos x+\sin x)(1-\cos x\cdot\sin x+\cos x\cdot\sin x) = \cos x+\sin x$

I

$\cos x+\sin x < 0 \quad \cos x < -\sin x$

$(\cos x+\sin x)^2 < -\sin 2x$

$1+\sin 2x < -\sin 2x$

$2\sin 2x < 1$

$\sin 2x < \frac{1}{2}$

$\frac{\pi}{} < 2x < \frac{7\pi}{6} + 2\pi n$

$-\frac{7\pi}{6} + 2\pi n < 2x < \frac{\pi}{6} + 2\pi n$

$-\frac{7\pi}{12} + \pi n < x < \frac{\pi}{12} + \pi n, \ n \in \mathbb{Z}$

просто ОДЗ

$\cos x + \sin x > 0$

$1 + \sin 2 < -\sin 2x$

$\sin 2x < \frac{1}{2}$

$2\pi n + \frac{7\pi}{6} < 2x < \frac{\pi}{6} + 2\pi n$

$\pi n + \frac{7\pi}{12} < x < \frac{\pi}{12} + \pi n$

$\sin\left(x+\frac{\pi}{4}\right) > 0$

$2\pi n < x + \frac{\pi}{4} < \pi + 2\pi n$

$-\frac{\pi}{4} + 2\pi n < x < \frac{3\pi}{4} + 2\pi n$