

ИДЛИКАР-ОЛА

02-84-73-58
(197.1)



Олимпиада ПБГ

2016

МОСКОВСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ
имени М.В.ЛОМОНОСОВА

Вариант 7-1

ПИСЬМЕННАЯ РАБОТА

Олимпиада школьников «Покори Воробьёвы горы!»

по МАТЕМАТИКЕ

Даниловой Анастасии Михайловны

фамилия, имя, отчество (в родительном падеже)

+1 месш Юл

Дата

«27» МАРТА 2016 года

Подпись участника

А

02-84-73-58
(197.1)

ЧИСТОВИК

1	2	3	4	5	7

$$\left(\frac{3}{2x-y} - \frac{2}{2x+y} - \frac{1}{2x-5y} \right) : \frac{y^2}{4x^2-y^2}$$

$$1) \frac{3(2x+y)(2x-5y) - 2(2x-y)(2x-5y) - (2x-y)(2x+y)}{(2x-y)(2x+y)(2x-5y)} =$$

$$= \frac{(6x+3y)(2x-5y) - 2(4x^2 - 10xy - 2xy + 5y^2) - (4x^2 - y^2)}{(4x^2 - y^2)(2x-5y)} =$$

$$= \frac{12x^2 - 30xy + 6xy - 15y^2 - 8x^2 + 24xy - 10y^2 - 4x^2 + y^2}{(4x^2 - y^2)(2x-5y)} =$$

$$= \frac{-24y^2}{(4x^2 - y^2)(2x-5y)}$$

$$2) \frac{-24y^2(4x^2 - y^2)}{(4x^2 - y^2)(2x-5y) \cdot y^2} = -\frac{24}{2x-5y}$$

при $x = \frac{4}{3}; y = \frac{4}{3}$

$$-\frac{24}{\frac{8}{3} - \frac{20}{3}} = -\frac{24 \cdot 3}{-2 \cdot 3} = \frac{24 \cdot 3}{6} = \frac{8}{3} = 2\frac{2}{3}$$

Ответ: $\frac{8}{3}$

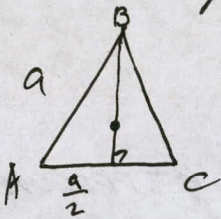
Верно

Решение верно

№4.

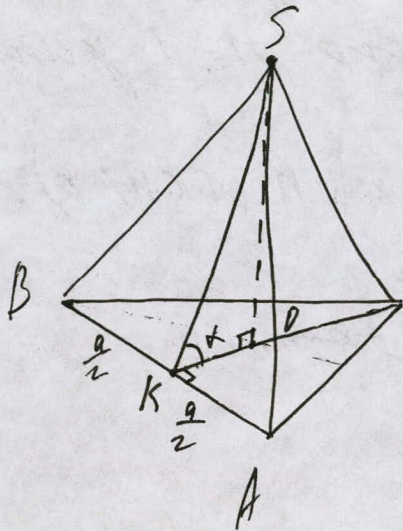
Реш: 1) т.к. центр сферы \in $\triangle ABC$, и т.ч. $A, B, C \in$ сфере \Rightarrow радиус сферы равен радиусу описан. круга $\triangle ABC$.

2) $\triangle ABC$ сторона $\neq a$.

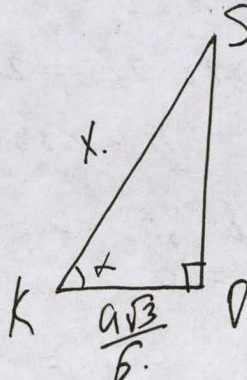


$$R = \frac{a\sqrt{3} \cdot 2}{2 \cdot 3} = \frac{a\sqrt{3}}{3} \text{ (радиус сферы)}$$

3)



т.к. пирамида правильная \Rightarrow двугран. уг. при основании — это $\angle SKC = \alpha$ $\Rightarrow \text{tg } \alpha = 3 \Rightarrow \text{tg } \angle SKC = 3$

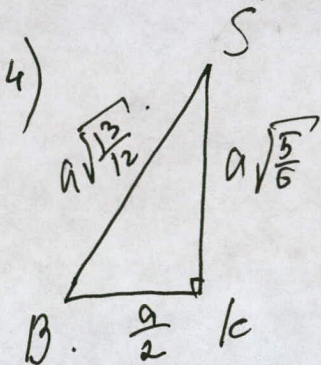


$$\text{tg}^2 \alpha + 1 = \frac{1}{\cos^2 \alpha}; \cos \alpha = \frac{KC}{SK} = \frac{a\sqrt{3}}{6x}$$

$$9 + 1 = \frac{36x^2}{a^2 \cdot 3}$$

$$5 \cdot 10a^2 = \frac{1}{6}x^2 \Rightarrow x = a\sqrt{\frac{5}{6}}$$

по т. Пиф. $SD = \sqrt{\frac{a^2 \cdot 5}{6} - \frac{a^2 \cdot 3}{36}} = \sqrt{\frac{28a^2}{36}} = \frac{a \cdot 2\sqrt{7}}{6} = \frac{a\sqrt{7}}{3}$



$$SB = \sqrt{\frac{a^2}{4} + \frac{a^2 \cdot 5}{6}} = \sqrt{\frac{13a^2}{12}} = a\sqrt{\frac{13}{12}}$$

№3.

Пусть изначально работало n учеников.

В плану каждой должен был покрасить $\frac{288}{n}$. Тогда мастер красит $\frac{288 \cdot 2,5}{n}$.

из усл., когда 1 заболел, кажд. ученик покрасил $\frac{288}{n} - 6$, и всего $n - 1$ учеников.

$$\left(\frac{288}{n} - 6\right)(n - 1) + \frac{288 \cdot 2,5}{n} = 288 \quad | \cdot n \text{ (т.к. } n \neq 0).$$

$$(288 - 6n)(n - 1) + 288 \cdot 2,5 = 288n.$$

$$288n - 6n^2 - 288 + 6n + 288 \cdot 2,5 = 288n \quad | :6.$$

$$n^2 - n - 48 \cdot 1,5 = 0$$

$$n^2 - n - 72 = 0.$$

$$\underline{\underline{n = 9}}$$

$$n = -8$$

т.к. n — кол-во учеников $\Rightarrow \Rightarrow \emptyset$.

изначально должно было работать 9 учеников.

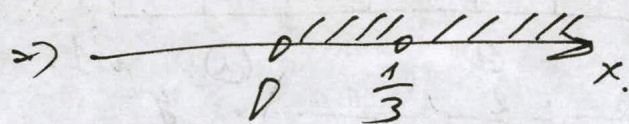
Ответ: 9.

верно

$$\log_{3x}(x+1) - (x+1) (\log_{\cos \theta} \sqrt{x+1})^{-1} < \sin^2 \theta.$$

$$\begin{cases} x+1 > 0 \\ 3x > 0 \\ 3x \neq 1 \\ x+1 \neq 0 \end{cases}$$

$$\Rightarrow \begin{cases} x > -1 \\ x > 0 \\ x \neq \frac{1}{3} \\ x > -1 \end{cases}$$



$$\log_{3x}(x+1) - (x+1) \log_{\cos \theta} \sqrt{x+1} < \sin^2 \theta.$$

$$\log_{3x}(x+1) - (x+1) \log_{\sqrt{x+1}} \cos \theta < \sin^2 \theta.$$

$$\log_{3x}(x+1) - \cos \theta \log_{\sqrt{x+1}}(x+1) < \sin^2 \theta.$$

$$\log_{3x}(x+1) - \cos^2 \theta < \sin^2 \theta.$$

$$\log_{3x}(x+1) < 1.$$

$$\frac{\ln(x+1) - \ln 3x}{\ln 3x - \ln 1} < 0$$

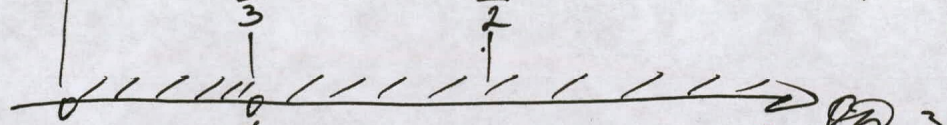
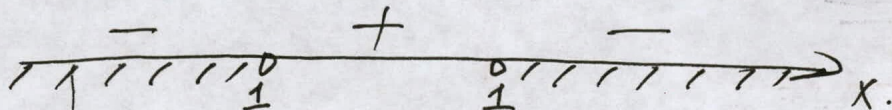
метод замены множителей.

$$\frac{x+1-3x}{3x-1} < 0.$$

$$\frac{-2x+1}{3x-1} < 0$$

$$н.ч. \neq \frac{1}{2}$$

$$н.з. x \neq \frac{1}{3}.$$



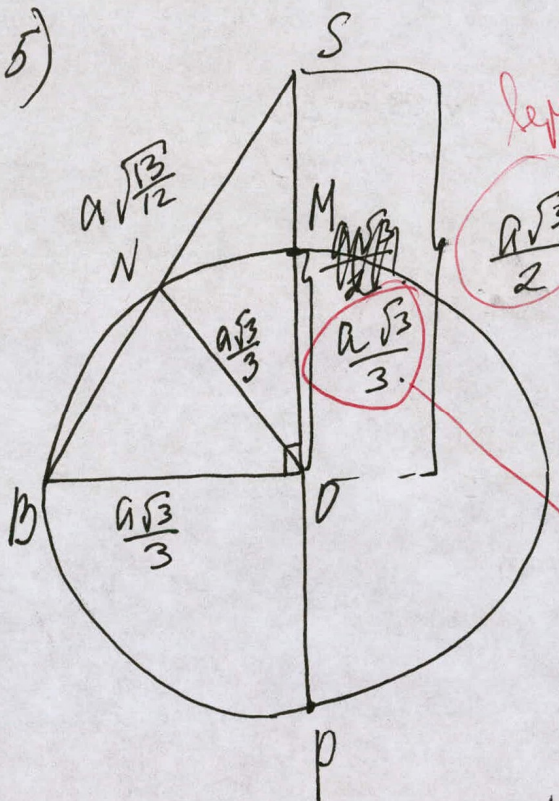
$$\text{Реш: } x \in (0, \frac{1}{3}) \cup (\frac{1}{2}, +\infty)$$

верно

ЧИСТОВИК

№4.

5)



верно

по св-ву секущих.

$$SM \cdot SP = SN \cdot SB$$

$$SM = \frac{a\sqrt{3}}{2} - \frac{a\sqrt{3}}{3} = a\sqrt{3} \left(\frac{1}{2} - \frac{1}{3} \right) =$$

$$= \frac{a\sqrt{3}}{6}$$

$$SP = \frac{a\sqrt{3}}{6} + 2 \cdot \frac{a\sqrt{3}}{3} = \frac{5a\sqrt{3}}{6}$$

верно

$$\frac{a\sqrt{3}}{6} \cdot \frac{5a\sqrt{3}}{6} = SN \cdot a\sqrt{\frac{13}{12}}$$

$$\frac{5 \cdot 3a}{6 \cdot 6} = SN \sqrt{\frac{13}{12}} \Rightarrow SN = \frac{5a \cdot \sqrt{12}}{12 \cdot \sqrt{13}} = \frac{5a}{\sqrt{12 \cdot 13}}$$

$$BN = a\sqrt{\frac{13}{12}} - \frac{5a}{\sqrt{12 \cdot 13}} = a \cdot \frac{(13-5)}{\sqrt{12 \cdot 13}} = \frac{a \cdot 8}{\sqrt{12 \cdot 13}} = \frac{a \cdot 4}{\sqrt{39}}$$

$$6) \frac{SN}{BN} = \frac{5 \cdot \sqrt{12 \cdot 13}}{\sqrt{12 \cdot 13} \cdot 4} = \frac{5}{4} = \frac{5}{8}$$

Ответ: $\frac{5}{8}$

верно

№5.

$$\cos(8\pi x) + 2\cos(4\pi x) - \cos(2\pi x) + 2\sin(\pi x) + 3 = 0.$$

замена $\pi x = t$.

$$\cos 8t + 2\cos 4t - \cos 2t + 2\sin t + 3 = 0.$$

нет решения

ЧЕРНОВИК

$$\frac{3}{2x-y} - \frac{2}{2x+y} - \frac{1}{2x-5y} =$$

$$= \frac{3(2x+y) - 2(2x-y) - 1}{(2x-y)(2x+y)(2x-5y)}$$

$$= \frac{3(2x+y)(2x-5y) - 2(2x-y)(2x-5y) - (2x-y)(2x+y)}{(2x-y)(2x+y)(2x-5y)}$$

$$= \frac{(6x+3y)(2x-5y) - (4x-2y)(2x-5y) - (4x^2-y^2)}{(2x-y)(4x^2-y^2)(2x-5y)}$$

$$= \frac{12x^2 - 30xy + 6xy - 15y^2 - 8x^2 + 20xy + 4xy - 10y^2 - 4x^2 + y^2}{(4x^2-y^2)(2x-5y)}$$

$$= \frac{-2x^2 - 24y^2}{(2x-5y)y^2}$$

$$x = \frac{4}{3} \quad y = \frac{4}{3}$$

$$= \frac{-\frac{2 \cdot 16}{9} - \frac{24 \cdot 49}{9}}{(2 \cdot \frac{4}{3} - 5 \cdot \frac{4}{3}) \cdot \frac{49}{9}}$$

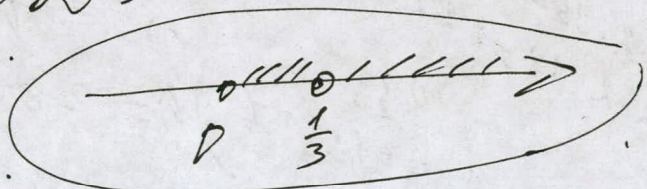
$$= \frac{(-32 - 24 \cdot 49) \cdot 3}{9(-28) \cdot 49}$$

$$\log_{3x}(x+1) - (x+1)^{\log_{\cos 9 \sqrt{x+1}}^{-1}} < \sin^2 9.$$

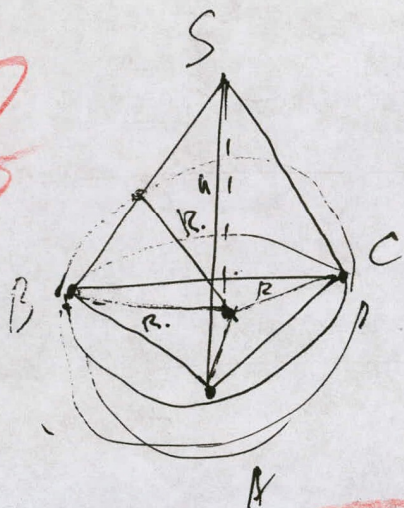
3:

$$\begin{cases} x+1 > 0 \\ 3x > 0 \\ 3x \neq 1 \\ \sqrt{x+1} > 0 \\ x+1 > 0 \\ x+1 > 0 \end{cases} \Rightarrow$$

$$\begin{cases} x > -1 \\ x > 0 \\ x \neq \frac{1}{3} \\ x > -1 \end{cases}$$



2



$$\sqrt{x^2+1} = \sqrt{x^2+1}^{\frac{1}{2}} = 2$$

$$\frac{1}{\log \cos g \sqrt{x+1}} = \log \sqrt{x+1} \cos g$$

$$(x+1) \log \sqrt{x+1} \cos g$$

$$= \cos g \log \sqrt{x+1} (x+1)$$

$$\log_{3x} - \cos^2 g < \sin^2 g = \cos^2 g$$

$$\log_{3x} < 1$$

$$2 \cos^2(4\pi x) - 1 + 2 \cos^2(2\pi x) - 1 - 2 \cos^2(\pi x) + 1 + 2 \sin(\pi x) + 3 = 0$$

$$2 (2 \cos^2(2\pi x) - 1)^2 - 1 + 4 (2 \cos^2(\pi x) - 1)^2 - 2 - 2 \cos(\pi x) + 1 + 2 \sin(\pi x) + 2 = 0$$

2/A

$$\cos 8y + 2 \cos 4y - \cos 2y + 2 \sin y + 3 = 0$$

$$2 \cos^2 4y - 1 + 2 \cos 4y - \cos 2y + 1 + 2 \sin y + 3 = 0$$

~~4+6~~

$$Xn = 288$$

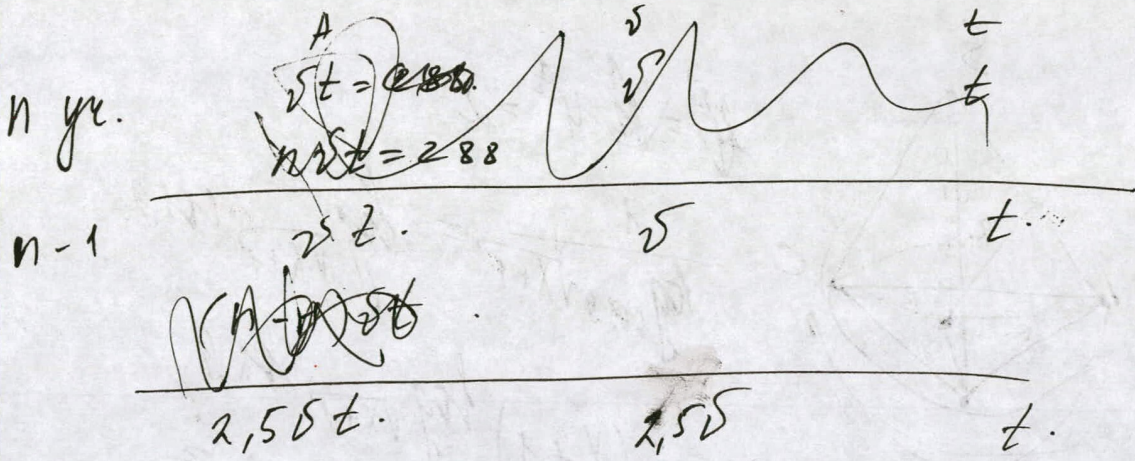
$$X = \frac{288}{n}$$

$$(X+6)(n-1) = 2,5X$$

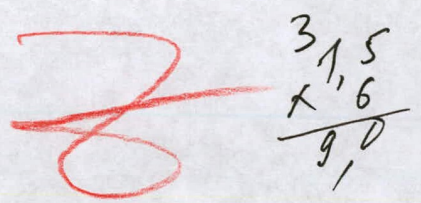
$$\frac{(288+6n)(n-1)}{n} = \frac{2,5 \cdot 288}{n}$$

$$288n + 6n^2 - 288 - 6n = 2,5 \cdot 288$$

$$6n^2 + 282n - 288 \cdot 3,5 = 0$$



$$\begin{cases} (n-1) \cdot 2.5t = 288 \\ (\sqrt{t} + 6)n = 288 \end{cases}$$



3,5
x 1,6

9,0
1

$$\begin{aligned} 2.5t(n-1+2.5) &= 288 \\ \sqrt{t} &= \frac{288}{n} - 6 = \frac{288-6n}{n} \end{aligned}$$

$$\frac{(288-6n)(n+1.5)}{n} = 288$$

$$288n - 6n^2 + 288 \cdot 1.5 - 9n = 288n$$

$$\begin{aligned} 2.5x &= 6(n-1) \\ x &= \frac{6n-6}{2.5} \end{aligned} \quad \begin{aligned} 2n^2 + 3n - 96 \cdot 1.5 &= 0 \\ 2n^2 + 3n - 144 &= 0 \end{aligned}$$

3 96
x 1,5

1 480
1 96

144,0

$$n = 2$$

$$n = 2$$

$$2 \cdot 2 \cdot 2 = 8 = 9 + 4 \cdot 2 \cdot 144$$

3 144
x 8

11 52
+ 9

1161

8 39
x 39

1351
1 x 7

8.5 21

8 29
x 29

261
5 2

8.4 1

8
x 39

1 351
117

1521

3 курс. дол. красит X.

$$(n-1)x + 2.5x = 288$$

$$x(n-1+2.5) = 288$$

$$(x+6)n = 288$$

$$x = \frac{288}{n+1.5}$$

$$\frac{(x+6) \cdot 288}{n+1.5} = 288$$

$$\begin{aligned} \left(\frac{288}{n+1.5} + 6\right)n &= 288 \\ \frac{(288 + 6n + 9)n}{n+1.5} &= 288 \end{aligned}$$

2,5
- 1,0

1,5

зам. $t = \sqrt{x+1} \Rightarrow x = t^2 - 1, t > 0.$

~~$3x = 3t - 3$~~ $t^2 = x + 1$
 $x = t^2 - 1$
 $3x = 3t^2 - 3.$

7

~~$\log_{(3t-3)} x = x$~~

$\log_{(3t^2-3)} t^2 - t^2 (\log_{\cos 9 t})^{-1} < \sin 9.$

УЗ.

7

288 \supset n - учетков.
 $n-1$ стало. $2 \sqrt{\text{проценты}}$

A	$\sqrt{\quad}$	t
\sqrt{t}	$\sqrt{\quad}$	t

~~$(5+6)n t = 288.$~~
 ~~$\sqrt{t} n + 2,5 \sqrt{t} = 288.$~~
 ~~$(5+6)n t \rightarrow 1.$~~

$(n-1) \sqrt{t} + 2,5 \sqrt{t} = 288.$

$\sqrt{t} (n-1+2,5) = 288$

$\sqrt{t} (n+1,5) = 288.$

~~$\sqrt{t} (n+2,5)$~~

~~$5n + 6n = \sqrt{n} + 2,5 \sqrt{n}$~~
 ~~$n = \frac{2,5 \sqrt{n}}{6}$~~

$(\sqrt{t} + 6)n = 288.$

$\frac{6(n-1)(n+1,5)}{2,5} = 288$

$(\frac{6(n-1)}{2,5} + 6)n = 288.$

$1 \cdot 2 \cdot 2 \cdot 2 =$
 $= 7 \cdot 2 \cdot 2 \cdot 36 =$
 $2 \cdot 2 \cdot 2 \cdot 2 \cdot 18 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3.$

$\sqrt{t} = \frac{288}{n+1,5}$

$2n^2 + 3n - 144 = 0$

$(\frac{288}{n+1,5} + 6)n = 288.$

$n = 2$

$n = 2$

$D = 9 + 4 \cdot 2 \cdot 144$

$(288 + 6n + 9)n = 288.$

$\frac{288n + 6n^2 + 9n}{n+1,5} = 288.$
 ~~$288n + 6n^2 + 9n = 288n + 288 \cdot 1,5.$~~
 $2n^2 + 3n = 288 \cdot 0,5.$
 $2n^2 + 3n = 144.$

ЧЕРНОВИК.

$$n^2 + 47n - 48 \cdot 3,5 = 0.$$

ОЛИМПИАДА

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2016

$$n^2 + 47n - 168 = 0$$

n =

n =

$$168 = 2 \cdot 84 = 2 \cdot 2 \cdot 42 = 2 \cdot 2 \cdot 2 \cdot 21 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 7.$$

$$\begin{array}{r} 48 \\ \times 3,5 \\ \hline 240 \\ 144 \\ \hline 168,0 \end{array}$$

$$\begin{cases} xn = 288 \\ 2,5x = 6(n-1) \end{cases} \quad x = \frac{288}{n}$$

$$\frac{2,5 \cdot 288}{n} = 6n - 6$$

$$\begin{array}{r} 48 \\ \times 2,5 \\ \hline 240 \\ 96 \\ \hline 120,0 \end{array}$$

$$2,5 \cdot 288 = 6n^2 - 6n$$

$$n^2 - n - 2,5 \cdot 48 = 0$$

$$n^2 - n - 120 = 0$$

$$D = 1 + 4 \cdot 120 = 481$$

$$\begin{array}{r} 21 \\ \times 21 \\ \hline 42 \\ 441 \\ \hline \end{array}$$

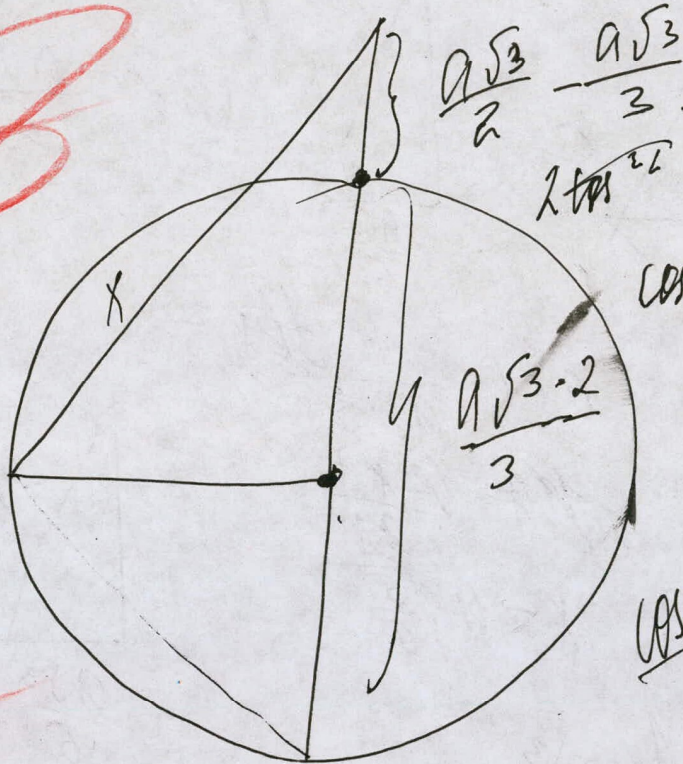
$$30 \cdot 4 = 120$$

$$x = \frac{288}{n} \quad b = \frac{288 - 6n}{n}$$

$$\begin{aligned} (b+n)n &= 288 \\ 2,5n &= 288 \\ (1,5+n)n &= 288 \end{aligned}$$

$$\frac{288 - 6n}{n} (1,5 + n) = 288$$

$$\frac{a \cdot 39}{36} - \frac{2ma\sqrt{39}}{3} + m^2 = a^2 \left(\frac{13}{12} - \sin \alpha \right)$$



$$\begin{aligned} \cos 2t &= 2\cos^2 t - 1 = \\ &= 2 - 2\sin^2 t - 1 = \\ &= 1 - 2\sin^2 t. \end{aligned}$$

$$\cos 4t = 1 - 2\sin^2 2t$$

$$\cos 8t = \cos 2t$$

$$\begin{aligned} \cos 8t &= 2\cos^2 4t - 1 = 2(2\cos^2 2t - 1)^2 - 1 = \\ &= 2(4\cos^4 2t - 4\cos^2 2t + 1) - 1 = \\ &= 8\cos^4 2t - 8\cos^2 2t + 1 = 0. \end{aligned}$$

$$8(2\cos^2 t - 1)^4 - 8(2\cos^2 t - 1)^2 + 1 = 0$$

~~cos~~

$$\cos x - \cos y = -2 \sin \left(\frac{x+y}{2} \right) \sin \left(\frac{x-y}{2} \right)$$

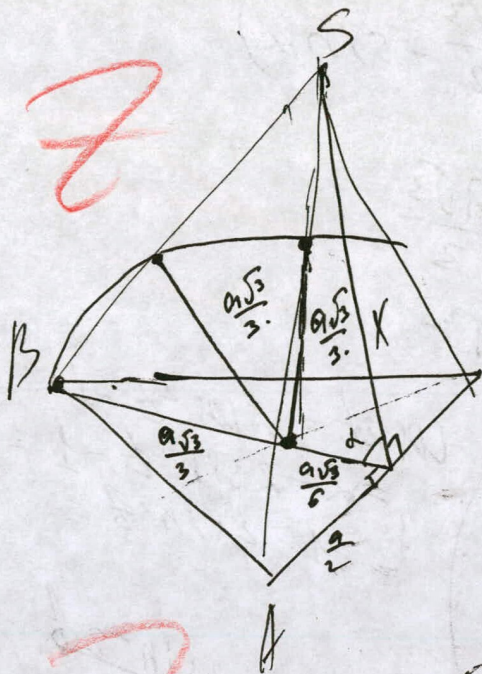
~~cos~~

$$\begin{aligned} \cos 8t - \cos 2t &= -2 \sin 5t \sin 3t = \\ &= -2 \sin(4t+t) \sin(2t+t) = \\ &= -2(\sin 4t \cos t + \cos 4t \sin t) (\sin 2t \cos t + \cos 2t \sin t). \end{aligned}$$

$$2\cos^2 4t - 1 + 2\cos 4t - 1 + 2\sin^2 t + 2\sin t + 3 = 0$$

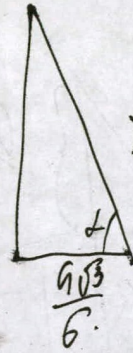
$$2\cos 4t (\cos 4t + 1) + 2\sin t (\sin t + 1) + 1 = 0$$

$$2(1 - 2\sin^2 2t)(1 - 2\sin^2 2t + 1) + 2\sin t (1 - 2\sin^2 t) (\cos^2 2t + 1) = 0$$



$\cos \alpha = \frac{1}{3}$

$\cos^2 \alpha + 1 = \frac{1}{\cos \alpha}$

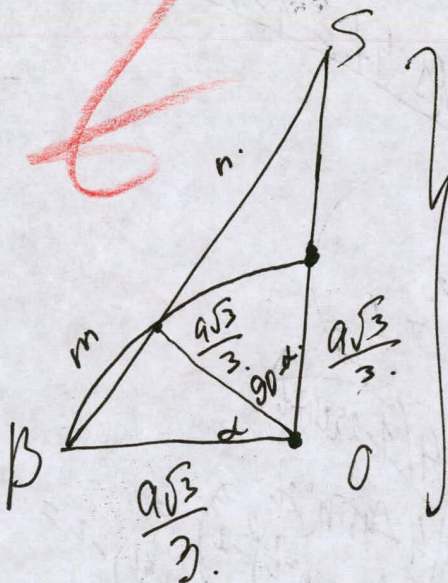
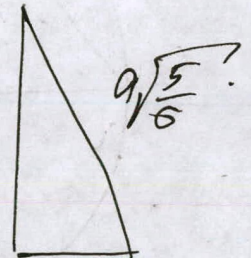


$\cos \alpha = \frac{a\sqrt{3}}{6x}$

$9 + 1 = \frac{9 \cdot 3 \cdot x^2}{a^2 \cdot 3}$

$5 \cdot 12x^2 = \frac{12x^2}{a^2}$

$x = a\sqrt{\frac{5}{6}}$



$\frac{a\sqrt{3}}{2}$

$\sqrt{\frac{a^2 \cdot 5}{6} - \frac{a^2 \cdot 3}{36}} = \frac{a \cdot 3 \sqrt{a^2 - 3a^2} = \frac{2\sqrt{a^2}}{36}$

$\frac{3a}{2} \sqrt{3} = \frac{a\sqrt{3}}{2}$

$SB = \sqrt{\frac{a^2 \cdot 3}{9} + \frac{a^2 \cdot 3}{4}} = \sqrt{\frac{12a^2 + 24a^2}{36}} = \frac{a}{6} \sqrt{39} = m + n$

$m^2 = \frac{a^2 \cdot 3}{9} + \frac{a^2 \cdot 3}{9} - 2 \cdot \frac{a^2 \cdot 3}{9} \cos \alpha = \frac{a^2 \cdot 3}{9} (2 - 2 \cos \alpha) = \frac{2a^2 \cdot 3}{9} (1 - \cos \alpha)$

$n^2 = \frac{a^2 \cdot 3}{9} + \frac{a^2 \cdot 3}{4} - 2 \cdot \frac{a\sqrt{3}}{3} \cdot \frac{a\sqrt{3}}{2} \cos(90 - \alpha) = \frac{3a^2}{36} - a^2 \sin \alpha = a^2 \left(\frac{1}{12} - \sin \alpha \right)$

$n = \frac{a\sqrt{39}}{6} - m$

$$X(n-1) + 2,5X = 288$$

$$(X+6)/n =$$

7

$$\begin{aligned} X(n-1) + 2,5X &= 288 \\ (X+6)n &= 288 \end{aligned}$$

манир, 200 карас манраас. $\frac{288}{n}$

$$\left(\frac{288}{n} - 6\right)(n-1) + \frac{288 \cdot 2,5}{n} = 288$$

$$\frac{(288-6n)(n-1) + 288 \cdot 2,5}{n} = 288n$$

$$288n - 6n^2 - 288 + 6n + 288 \cdot 2,5 = 288n$$

$$6n^2 - 6n - 288 \cdot 1,5 = 0$$

$$n^2 - n - 48 \cdot 1,5 = 0$$

$$n^2 - n - 72 = 0$$

$$n = 9$$

$$n = -8$$

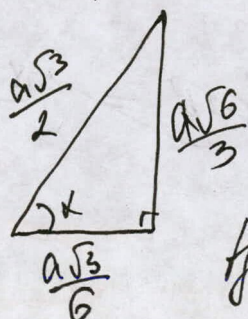
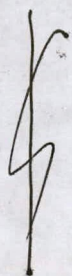
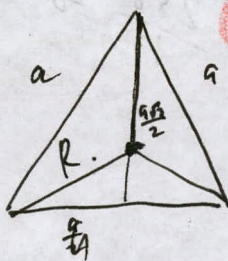
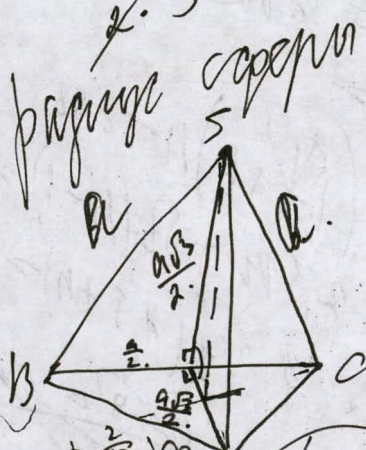
$$\begin{array}{r} 448 \\ \times 1,5 \\ \hline 1240 \\ 48 \\ \hline 720 \end{array}$$

$$2 \cdot 36 = 2 \cdot 2 \cdot 9 = 2 \cdot 2 \cdot 3 \cdot 3$$

$$2 \cdot 2 \cdot 18 = 2 \cdot 2 \cdot 2 \cdot 9$$

$$x = \sqrt{\frac{a^2 \cdot 3}{4} - \frac{a^2 \cdot 3}{36}} = \frac{a\sqrt{3}}{2} \sqrt{1 - \frac{1}{9}} = \frac{a\sqrt{3}}{2} \cdot \frac{2\sqrt{2}}{3}$$

$$\frac{a\sqrt{3} \cdot 2}{2 \cdot 3} = \frac{a\sqrt{3}}{3} = R$$



$$R = 3 \cdot \frac{a\sqrt{3}}{6} = \frac{a\sqrt{3}}{2}$$